

Lie algebras and algebras of associative type

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Abstract

In the paper, some properties of algebras of associative type are studied, and these properties are then used to describe the structure of finite-dimensional semisimple modular Lie algebras. It is proved that the homogeneous radical of any finite-dimensional algebra of associative type coincides with the kernel of some form induced by the trace function with values in a polynomial ring. This fact is used to show that every finite-dimensional semisimple algebra of associative type $A = \bigoplus_{\alpha \in G} A_{\alpha}$ graded by some group G , over a field of characteristic zero, has a nonzero component A_1 (where 1 stands for the identity element of G), and A_1 is a semisimple associative algebra. Let $B = \bigoplus_{\alpha \in G} B_{\alpha}$ be a finite-dimensional semisimple Lie algebra over a prime field F_p , and let B be graded by a commutative group G . If $B = F_p \otimes_{\mathbb{Z}} A_L$, where A_L is the commutator algebra of a \mathbb{Z} -algebra $A = \bigoplus_{\alpha \in G} A_{\alpha}$; if $\mathbb{Q} \otimes_{\mathbb{Z}} A$ is an algebra of associative type, then the 1 -component of the algebra $K \otimes_{\mathbb{Z}} B$, where K stands for the algebraic closure of the field F_p , is the sum of some algebras of the form $gl(n_i, K)$. © 2010 Pleiades Publishing, Ltd.

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Keywords

algebra of associative type, finite-dimensional semisimple algebra of associative type, finite-dimensional semisimple Lie algebra, graded algebra